Vu Thi Huong's Report on "Postes Rouges CNRS 2023"

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Dates of stay: 02/09/2023 – 29/11/2023

Proposed Project: Some Selected Problems in Variational Analysis and Applications
The project aims at studying calculus rules for coderivatives of convex set-valued maps by a metric approach and its applications. The expected result is a manuscript to be published on a peer-reviewed international journal.

Achievements:

- 01 seminar talk entitled "The split feasibility problem and beyond" was delivered within the working group "Statistics, Probability, Optimization and Control" of the Burgundy Institute of Mathematics, University of Burgundy on October 25, 2023. (https://indico.math.cnrs.fr/event/10833/)

  Abstract of the talk: Given nonempty, closed and convex sets $C \subset \mathcal{H}$, $Q \subset \mathcal{K}$ in Hilbert spaces $\mathcal{H}, \mathcal{K}$ and a bounded linear operator $A : \mathcal{H} \to \mathcal{K}$, the *split feasibility problem* (SFP) is to find $x \in C$ such that $Ax \in Q$. The problem was introduced by Censor and Elfving [Numer. Algorithms 1994] to model phase retrieval and other image restoration problems in signal processing. During the last three decades, many efforts have been made to design solution algorithms for SFP. Interestingly, this feasibility problem can be reformulated as a fixed point problem or a convex minimization one; hence, advanced tools from operator theory, convex analysis, and optimization machinery can be fully exploited. In the first part of the talk, we will review some basic solution algorithms for SFP resulting from this approach, and then discuss further the gradient projection method with Polyak's stepsize. The second part of the talk is about solution stability of SFP with respect to small changes of input data, where SFP is reformulated as a parametric generalized equation to which set-valued and variational analysis techniques are applied. The talk is based on several recent joint papers by Prof. Hong Kun Xu (Hangzhou Dianzi University), Prof. Nguyen Dong Yen (Institute of Mathematics, Vietnam Academy of Science and Technology), and the speaker.

- 01 manuscript in progress: Vu Thi Huong and Abderrahim Jourani, Discrete optimal

control with disturbances: from coderivative theory to computational aspects[1], 2023. (working, 24 pages)

This work is about coderivative calculus and its applications in optimal control. Namely, we consider discrete optimal control problems which contain disturbances, i.e., external factors of control systems with values beyond the designer's control, as parameters. Various stability properties of the control problem with respect to parameters' perturbation can be obtained by exploiting advanced tools from variational analysis, including results on coderivative calculus for set-valued maps. In order to apply such a result, constraint qualification conditions are usually needed. However, there might be challenges arising from validating constraint qualification conditions. This issue is addressed from computational aspects and then motivates us to study a special but popular class of optimal control problems where constraint qualification conditions can be dropped. Among other things, it is interesting that the split feasibility problem, introduced by Censor and Elfving [Numer. Algorithms 1994] to model image restoration problems in signal processing, arises for the first time in optimal control, as a sub problem in our proposed algorithms for detecting irregular points of linear control systems.

The report is written by Vu Thi Huong on November 19, 2023. Below is a picture of her taken in the beautiful center of Dijon, in November, 2023.

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[1]The title is subject to change, because more results might still come.