Topological realization of certain resolution of the singular cohomology of $\mathrm{BO}(2)$

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Quasi-resolution

$$\mathrm{Ext}^*_{\mathscr{C}}(M, N)$$

Quasi-resolution

$$\mathrm{Ext}^{*}_{\Bbbk G}(\Bbbk, \Bbbk)$$

Quasi-resolution

$$\mathrm{Ext}_{\mathscr{A}}^{*,*}(\mathbb{F}_p, \mathbb{F}_p)$$

Quasi-resolution

$$\mathrm{Ext}_{\mathscr{U}}^{*,*}(\mathbb{F}_p, \mathbb{F}_p)$$

Quasi-resolution

$$\mathrm{Ext}^*_{\mathscr{C}}(M, N)$$

NEC

$$N \longrightarrow N^0 \longrightarrow N^1 \longrightarrow N^2 \longrightarrow N^3 \longrightarrow N^4 \longrightarrow \cdots$$

$$
\begin{array}{ccccccccccc}
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
 & & \mathscr{I}^{0,2} & & \mathscr{I}^{1,2} & & \mathscr{I}^{2,2} & & \mathscr{I}^{3,2} & & \mathscr{I}^{4,2} & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
 & & \mathscr{I}^{0,1} & & \mathscr{I}^{1,1} & & \mathscr{I}^{2,1} & & \mathscr{I}^{3,1} & & \mathscr{I}^{4,1} & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
 & & \mathscr{I}^{0,0} & & \mathscr{I}^{1,0} & & \mathscr{I}^{2,0} & & \mathscr{I}^{3,0} & & \mathscr{I}^{4,0} & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
\mathrm{N} & \longrightarrow & \mathrm{N}^0 & \longrightarrow & \mathrm{N}^1 & \longrightarrow & \mathrm{N}^2 & \longrightarrow & \mathrm{N}^3 & \longrightarrow & \mathrm{N}^4 & \longrightarrow & \cdots
\end{array}
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 & & \mathscr{I}^{0,1} & \longrightarrow & \mathscr{I}^{1,1} & \longrightarrow & \mathscr{I}^{2,1} & \longrightarrow & \mathscr{I}^{3,1} & \longrightarrow & \mathscr{I}^{4,1} & \longrightarrow & \cdots \\
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 & & \mathscr{I}^{0,0} & \longrightarrow & \mathscr{I}^{1,0} & \longrightarrow & \mathscr{I}^{2,0} & \longrightarrow & \mathscr{I}^{3,0} & \longrightarrow & \mathscr{I}^{4,0} & \longrightarrow & \cdots \\
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 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
N & \longrightarrow & N^0 & \longrightarrow & N^1 & \longrightarrow & N^2 & \longrightarrow & N^3 & \longrightarrow & N^4 & \longrightarrow & \cdots
\end{array}
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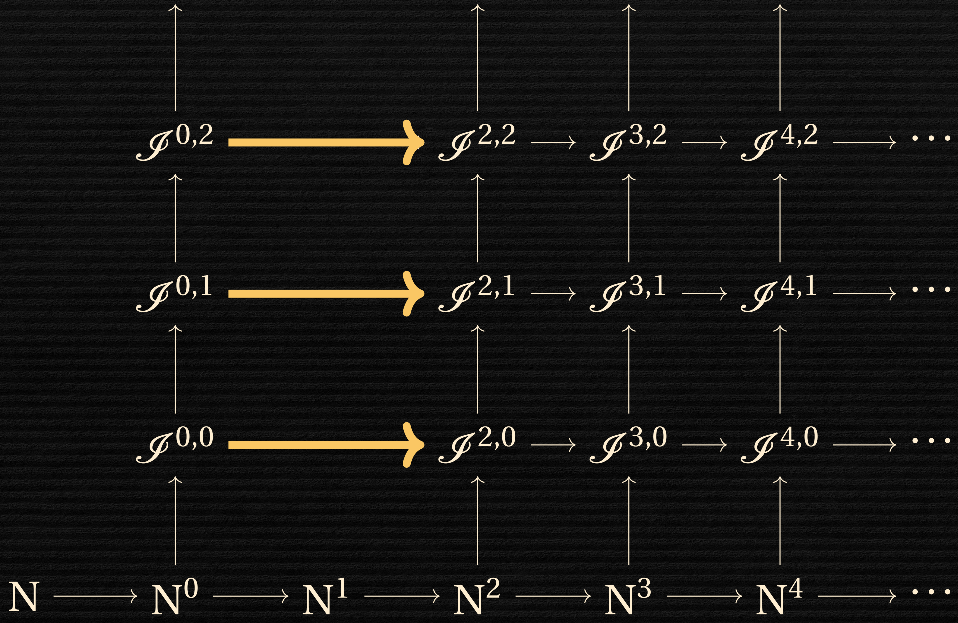

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N N^0 N^1 N^2 N^3 N^4 $\cdots$

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\begin{array}{ccccccccccc}
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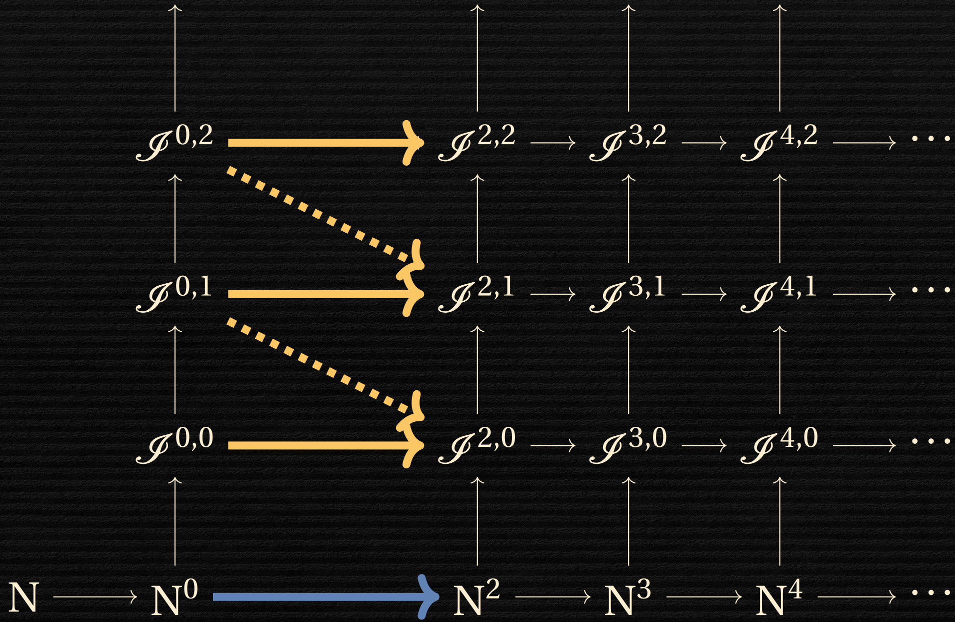
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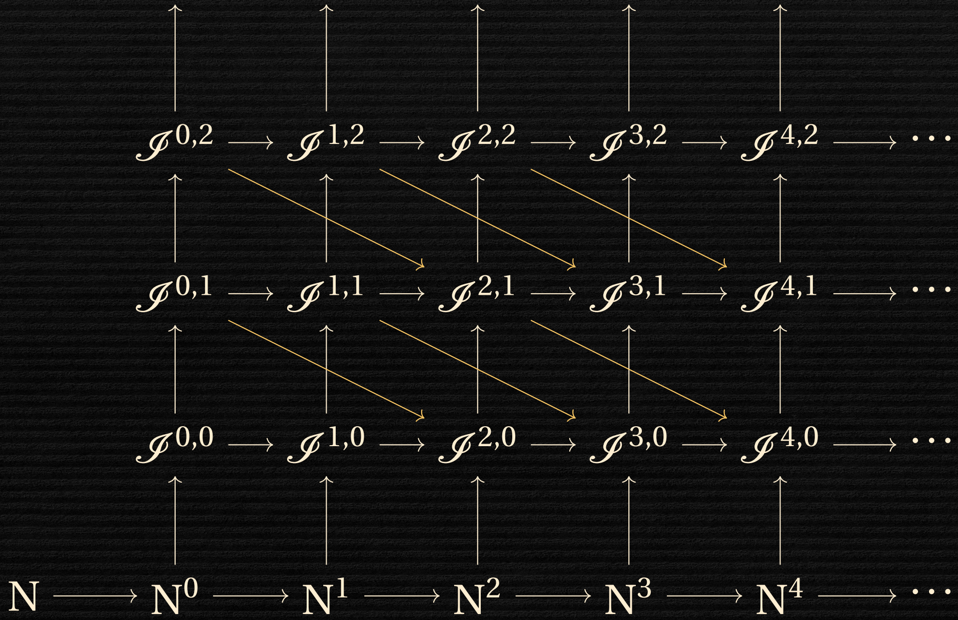
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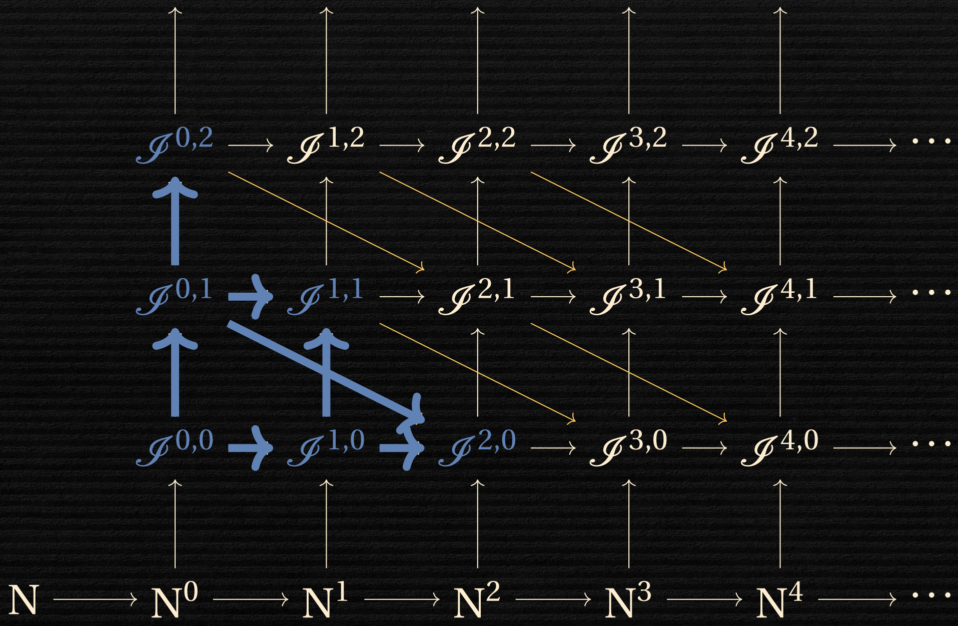
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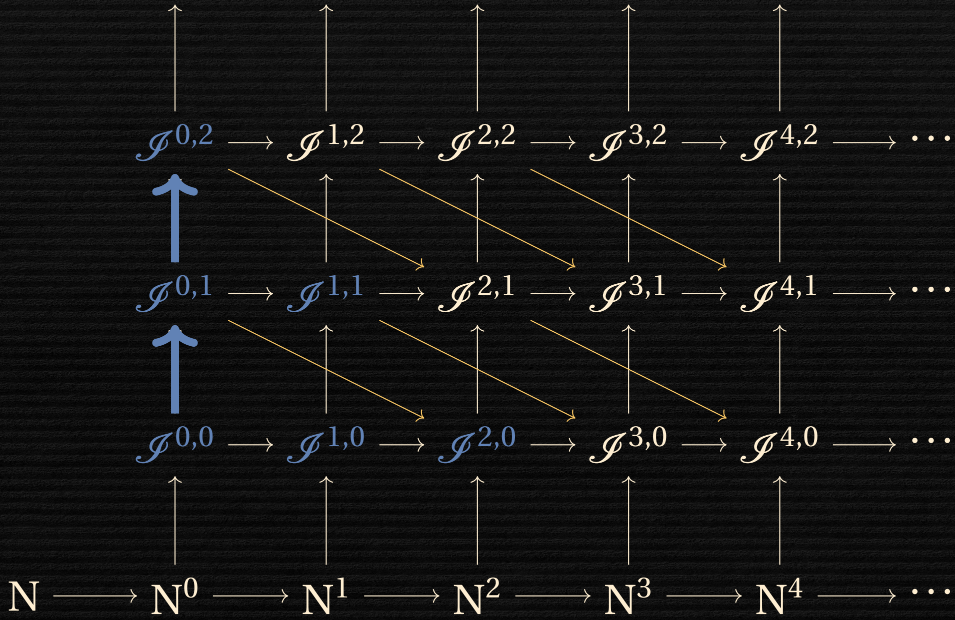
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$\mathscr{I}^{0,0}$ $\mathscr{I}^{1,0}$ $\mathscr{I}^{2,0}$ $\mathscr{I}^{3,0}$ $\mathscr{I}^{4,0}$ $\cdots$
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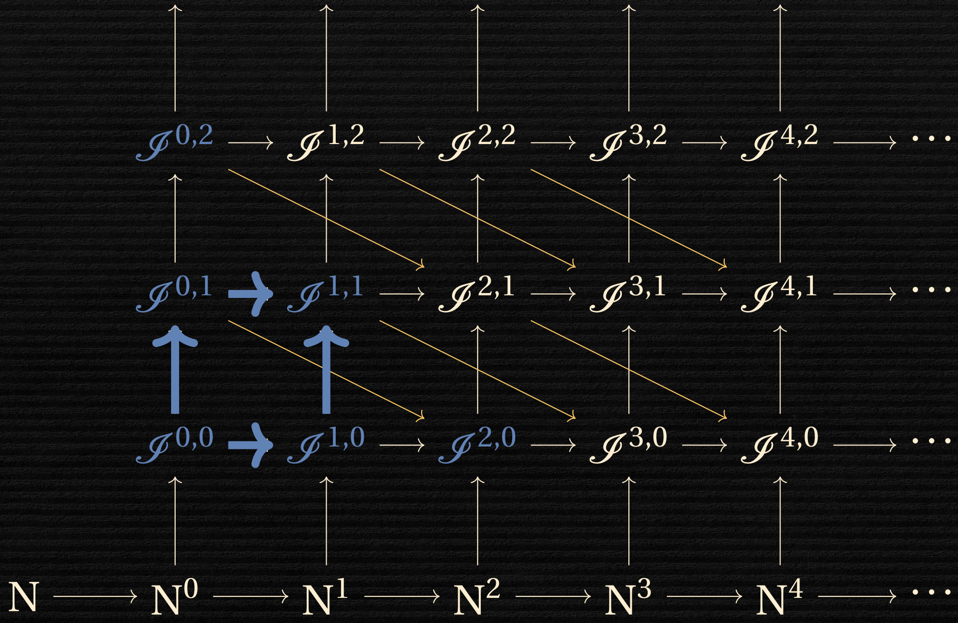

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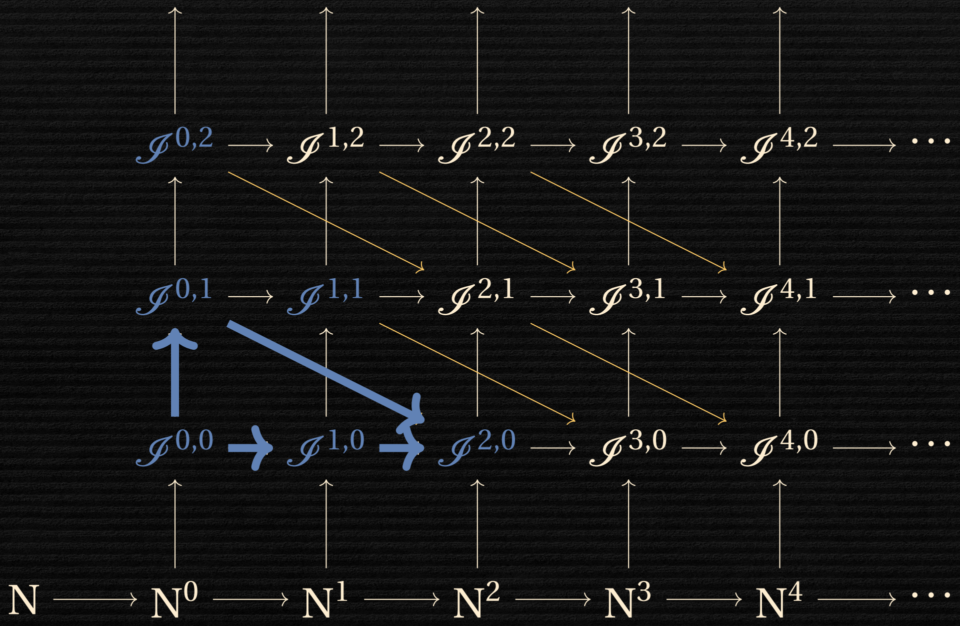
𝒥^{0,0} → 𝒥^{1,0} → 𝒥^{2,0} → 𝒥^{3,0} → 𝒥^{4,0} → ···
N → N^0 → N^1 → N^2 → N^3 → N^4 → ···



𝒥^{0,2} 𝒥^{1,2} 𝒥^{2,2} 𝒥^{3,2} 𝒥^{4,2} ⋯
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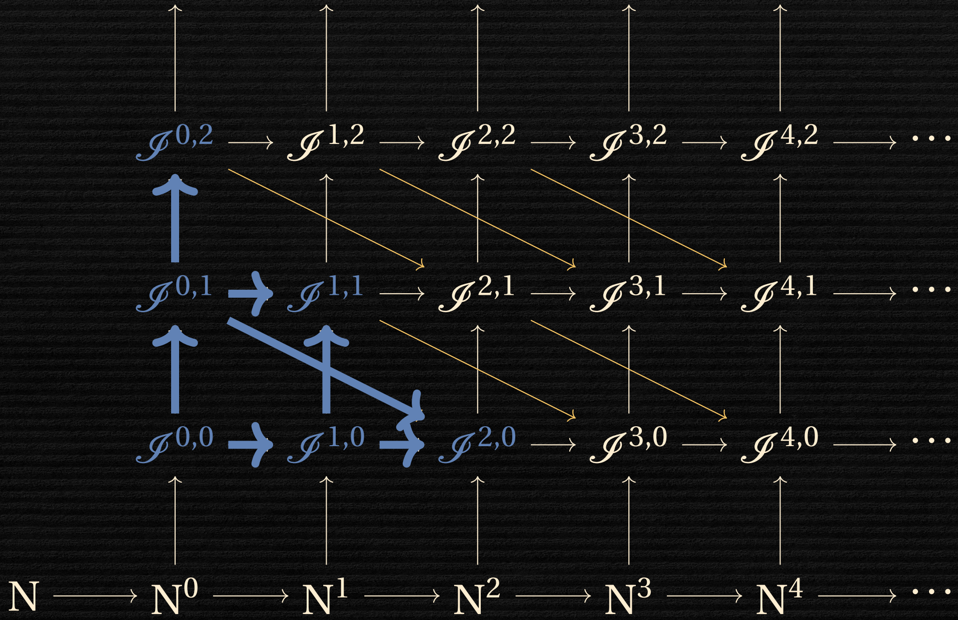

$\mathscr{I}^{0,2}$ $\mathscr{I}^{1,2}$ $\mathscr{I}^{2,2}$ $\mathscr{I}^{3,2}$ $\mathscr{I}^{4,2}$ $\cdots$
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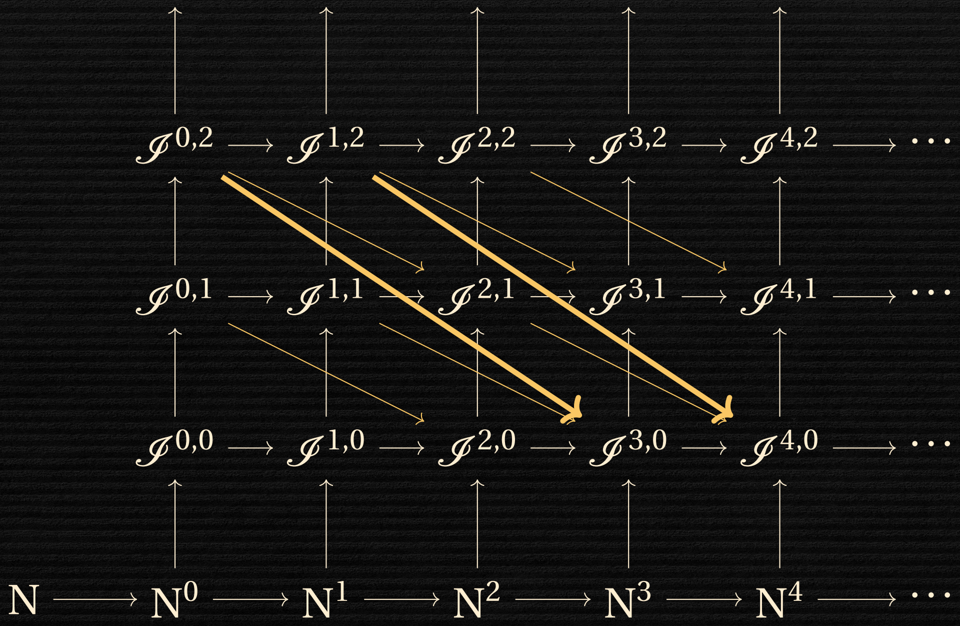
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N N^0 N^1 N^2 N^3 N^4 $\cdots$

Theorem

Let $\mathscr{C}$ be an abelian category with enough injective objects. Let $\mathrm{N} \in \mathscr{C}$ and let $(\mathrm{N}^\bullet)$ be a resolution of N. For each $k \geq 0$, let $\left(\mathscr{I}^{k,\bullet}\right)$ be an injective resolution of N^k. Then, there exists an injective resolution $(\mathrm{J}^\bullet)$ of N such that:

$$\mathrm{J}^n = \bigoplus_{i+j=n} \mathscr{I}^{i,j}.$$

This resolution is called the quasi-resolution of N with respect to the collection of injective resolutions $(\mathscr{I}^{\bullet,\bullet})$.

$$H = H^{*}(\mathbb{R}P^{\infty}; \mathbb{Z}/2)$$

$$H^{*}BS^{1} \longrightarrow H \longrightarrow \Sigma H \longrightarrow \Sigma^{2} H \longrightarrow \Sigma^{3} H \longrightarrow \Sigma^{4} H$$

$$
\begin{array}{cccccc}
 & 0 & 0 & 0 & 0 & J(1)\otimes H \\
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
 & 0 & 0 & 0 & J(1)\otimes H & J(2)\otimes H \\
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
 & 0 & 0 & J(1)\otimes H & J(2)\otimes H & (J(3)\oplus J(2))\otimes H \\
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
 & H & \Sigma H & J(2)\otimes H & J(3)\otimes H & J(4)\otimes H \\
 & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
H^*BS^1 \longrightarrow & H \longrightarrow & \Sigma H \longrightarrow & \Sigma^2 H \longrightarrow & \Sigma^3 H \longrightarrow & \Sigma^4 H
\end{array}
$$

$$
\begin{array}{ccccccccccc}
 & & 0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & J(1)\otimes H \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 & & 0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & J(1)\otimes H & \longrightarrow & J(2)\otimes H \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 & & 0 & \longrightarrow & 0 & \longrightarrow & J(1)\otimes H & \longrightarrow & J(2)\otimes H & \longrightarrow & (J(3)\oplus J(2))\otimes H \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & \searrow & \uparrow \\
 & & H & \longrightarrow & \Sigma H & \longrightarrow & J(2)\otimes H & \longrightarrow & J(3)\otimes H & \longrightarrow & J(4)\otimes H \\
 & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
H^{*}BS^{1} & \longrightarrow & H & \longrightarrow & \Sigma H & \longrightarrow & \Sigma^{2}H & \longrightarrow & \Sigma^{3}H & \longrightarrow & \Sigma^{4}H
\end{array}
$$

$$O(2)$$

$$\|$$

$$\left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{R}) \,\middle|\, A \cdot A^t = I_2 \right\}$$

$$\left\langle \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle \subset O(2)$$

$$\|$$

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$$\left(\mathbb{Z}/_{2}\right)^2$$

$$\cong$$

$$\left\langle \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\rangle \subset \mathrm{O}(2)$$

$$\|$$

$$\left\{ \mathrm{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{M}_2(\mathbb{R}) \,\middle|\, \mathrm{A} \cdot \mathrm{A}^t = \mathrm{I}_2 \right\}$$

$$
\begin{aligned}
H^*BO(2) &\cong \mathbb{F}_2[x+y, xy] \\
&\cong \Gamma^2(H)
\end{aligned}
$$

$$\mathrm{O}(2) \xrightarrow{\det} \mathbb{Z}/2$$

$$\mathrm{SO}(2) \hookrightarrow \mathrm{O}(2) \xrightarrow{\det} \mathbb{Z}/2$$

$$\mathrm{SO}(2) \hookrightarrow \mathrm{O}(2) \xrightarrow{\det} \mathbb{Z}/2$$

$$\cong$$

$$S^1$$

Exact sequence

$$0 \longrightarrow H^*BO(2) \xrightarrow{\text{Res}} H^*B\left(\mathbb{Z}/_2\right)^{\oplus 2} \xrightarrow{\text{Tr}} H^*BO(2) \xrightarrow{\text{Res}} H^*BS^1 \longrightarrow 0$$

Exact sequence

$$
\begin{array}{ccccccccccc}
0 & \longrightarrow & H^*BO(2) & \xrightarrow{\text{Res}} & H^*B\left(\mathbb{Z}/2\right)^{\oplus 2} & \xrightarrow{\text{Tr}} & H^*BO(2) & \xrightarrow{\text{Res}} & H^*BS^1 & \longrightarrow & 0 \\
 & & \| & & \| & & \| & & \| & & \\
0 & \longrightarrow & \Gamma^2(H) & \hookrightarrow & H \otimes H & \xrightarrow{1+\sigma} & \Gamma^2(H) & \longrightarrow & \Phi H & \longrightarrow & 0
\end{array}
$$

$$\Gamma^2(H) \longrightarrow H \otimes H \longrightarrow \Gamma^2(H) \longrightarrow \Phi H \longrightarrow 0$$

$$
\begin{array}{ccccccccc}
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
 & & H \otimes H & & & & & & \\
 & & \uparrow & & & & & & \\
\Gamma^2(H) & \longrightarrow & H \otimes H & \longrightarrow & \Gamma^2(H) & \longrightarrow & \Phi H & \longrightarrow & 0
\end{array}
$$

$$
\begin{array}{ccccccccc}
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
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 & & \uparrow & & & & & & \\
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
 & & H \otimes H & & H \otimes H & & & & \\
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 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
 & & 0 & & & & & & \\
 & & \uparrow & & & & & & \\
 & & H \otimes H & & H \otimes H & & & & \\
 & & \uparrow & & \uparrow & & & & \\
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 & & 0 & & & & & & \\
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\Gamma^2(H) & \longrightarrow & H \otimes H & \longrightarrow & \Gamma^2(H) & \longrightarrow & \Phi H & \longrightarrow & 0
\end{array}
$$

$$
\begin{array}{ccccccccc}
 & & 0 & & & & (J(3) \oplus J(1)) \otimes H & & \\
 & & \uparrow & & & & \uparrow & & \\
 & & 0 & & & & J(2) \otimes H & & \\
 & & \uparrow & & & & \uparrow & & \\
 & & 0 & & H \otimes H & & \Sigma H & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & H \otimes H & & H \otimes H & & H & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
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 & & \uparrow & & & & \uparrow & & \\
 & & 0 & & H \otimes H & & \Sigma H & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & H \otimes H & & H \otimes H & & H & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
\Gamma^2(H) & \longrightarrow & H \otimes H & \longrightarrow & \Gamma^2(H) & \longrightarrow & \Phi H & \longrightarrow & 0
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 & & 0 & & H \otimes H \oplus H & & J(2) \otimes H & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & 0 & & H \otimes H & & \Sigma H & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
 & & H \otimes H & & H \otimes H & & H & & \\
 & & \uparrow & & \uparrow & & \uparrow & & \\
\Gamma^2(H) & \longrightarrow & H \otimes H & \longrightarrow & \Gamma^2(H) & \longrightarrow & \Phi H & \longrightarrow & 0
\end{array}
$$

Resolution of $\Gamma^2(H)$

$$H \otimes H \xrightarrow{\partial^0} H \otimes H \xrightarrow{\partial^1} \begin{matrix} H \otimes H \\ \oplus \\ R(0) \otimes H \end{matrix} \xrightarrow{\partial^2} \begin{matrix} H \otimes H \\ \oplus \\ R(1) \otimes H \end{matrix} \xrightarrow{\partial^3} \begin{matrix} H \otimes H \\ \oplus \\ R(2) \otimes H \end{matrix} \xrightarrow{\partial^4} \cdots$$

Resolution of $\Gamma^2(H)$

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$$\partial^0 = 1 + \sigma,$$

Resolution of $\Gamma^2(H)$

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$$\partial^0 = 1 + \sigma,$$

$$\partial^1 = \begin{bmatrix} 1 + \sigma \\ \mathrm{mult} \end{bmatrix},$$

Resolution of $\Gamma^2(H)$

$$H \otimes H \xrightarrow{\partial^0} H \otimes H \xrightarrow{\partial^1} \begin{matrix} H \otimes H \\ \oplus \\ R(0) \otimes H \end{matrix} \xrightarrow{\partial^2} \begin{matrix} H \otimes H \\ \oplus \\ R(1) \otimes H \end{matrix} \xrightarrow{\partial^3} \begin{matrix} H \otimes H \\ \oplus \\ R(2) \otimes H \end{matrix} \xrightarrow{\partial^4} \cdots$$

$$\partial^0 = 1 + \sigma,$$

$$\partial^1 = \begin{bmatrix} 1 + \sigma \\ \text{mult} \end{bmatrix},$$

$$\partial^k = \begin{bmatrix} 1 + \sigma & 0 \\ (pr_{k-1} \otimes id_H) \circ \alpha & \beta^{k-2} \end{bmatrix}$$

Algebraic connection

$$\mathrm{Ext}^{s}_{\mathscr{U}}\left(\Sigma^{n}\mathbb{F}_2, \Sigma^{t}\mathrm{H}^{*}\left(\mathrm{BO}\left(2\right)\right)\right)$$

$$\cong$$

$$\mathrm{Ext}^{s}_{\mathscr{U}}\left(\Sigma^{n}\mathbb{F}_2, \mathrm{H}^{*}\left(\mathrm{S}^{t}\right)\right) \oplus \left[\bigoplus_{\substack{a+b=s\\ b\geq 3}} \mathrm{Ext}^{a}_{\mathscr{U}}\left(\Sigma^{n}\mathbb{F}_2, \tilde{\mathrm{H}}^{*}\left(\Sigma^{t}\mathbb{R}\mathrm{P}^{b-2}\right)\right)\right]$$

THANK YOU